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MEETING OF THE MINNESOTA SECTION.

The annual spring meeting of the Minnesota Section was held on Saturday, May 31, 1919, at Science Hall, Hamline University. There were thirty-one present including the following members of the Association: R. M. Barton, W. O. Beal, Edla G. Berger, W. H. Bussey, H. H. Dalaker, C. H. Gingrich, R. A. Johnson, Sister Mary John (institutional representative), R. M. Mathews, C. A. V. Peterson, Jessie G. Quigley, W. D. Reeve, Ella A. M. Thorp, C. H. Yeaton.

The program consisted of chairman's remarks by C. H. Gingrich, Carleton College, a paper by W. F. Swann, of the department of physics, University of Minnesota, a book review by W. O. Beal, of the department of astronomy, University of Minnesota, a memorial tribute to Father William Earnshaw Etzel by Father Moynihan, President of St. Thomas College, a discussion of college entrance requirements for mathematics opened by W. D. Reeve, College of Education, University of Minnesota, popular talks on mathematics by Jessie G. Quigley, College of St. Teresa, and five papers upon mathematical subjects by R. M. Mathews, Duluth Central High School, W. H. Kirchner of the college of engineering, University of Minnesota, R. A. Johnson, Hamline University, D. C. Kazarinoff, Carleton College, and W. H. Bussey, University of Minnesota.

The program was unusually well selected and presented and all papers proved of interest to the audience. Mr. Reeve, in opening the discussion of college entrance requirements in mathematics, urged the use of mental tests in addition to the method of certification and presented a considerable body of data which has grown out of his experience in this work.

Mr. Mathews's paper on "The graphics of intersections of line and conic" appears elsewhere in this issue of the Monthly.

Mr. Kirchner followed his discussion of last year upon the desirability of introducing Descriptive Geometry as a course in college mathematics, with a short paper upon "The co-point of three planes" in which he set up the conditions for the common point of three planes from the point of view of descriptive geometry and interpreted these in terms of the corresponding conditions from the view points of analytic geometry and algebra.

Dr. Johnson's paper dealt with the well-known theorem usually ascribed to

Fontené: The necessary and sufficient condition that the pedal circle of a point be tangent to the nine-point circle in that the point and its isogonal conjugate be collinear with the circumcenter. An alternative method of finding, with ruler and compasses, the intersections of a line with a conic given by five points was incidentally developed.

Mr. Kazarinoff's paper on "Some properties of focal conics of quadrics" appears elsewhere in this issue of the MONTHLY as "Dupin's Theorem."

President Moynihan of the College of St. Thomas related the events of the life of Father William Earnshaw Etzel in a very interesting manner, giving an insight into the motives and accomplishments of Father Etzel at once keen and sympathetic. Father Etzel was a charter member of the Section, an officer and an ardent helpful worker whose presence is greatly missed.

Mr. Swann's paper on "The diurnal variation of the atmospheric potential-gradient" is in abstract as follows:

Various phenomena in cosmical physics lend support to the view that the upper atmosphere is highly conducting as compared with the air near the earth. Thus, for example, considerations based on the ionization of the upper atmosphere by the sun's ultra-violet light show that this agency is capable of accounting for such a high conductivity that the resistance of a column of the upper air equal in length to the earth's quadrant would be no more than that of a column 3 cms. long and of the same cross section at the earth's surface. Thus, as far as this agency is concerned, the upper atmosphere may, for electrostatic purposes, be looked upon as a perfect conductor. Although the ultra violet light affects only the sunlit portions of the atmosphere, we have reason to believe that the corpuscular radiation responsible for the aurora is more evenly distributed.

If two concentric spheres be maintained at a difference of potential, the field at the surface of the inner sphere will, from symmetry, be the same at all points. If a dent be made in the outer sphere, the distance between the spheres at this point will be decreased, and since each sphere is at the same potential at all points, the *field* at the portion of the surface of the inner sphere under the dent will be stronger than the field elsewhere. This crude illustration suggests the view that if the upper atmosphere is in a highly conducting state, but the high conductivity on the sunlit side extends to lower levels than on the opposite side, the potential gradient should be higher in the former case than in the latter.

Here then would be an influence playing a part in the determination of the diurnal variation of the potential gradient, and such as to predict a maximum by day and a minimum by night, which condition is found, in a general way, to prevail. In its simplest aspects it would predict a diurnal variation of the air-earth conduction current-density of the same type as the potential-gradient. In practice it is found that the potential-gradient and conductivity at the earth's surface vary in opposite ways in such a manner as to secure greater constancy in the conduction current-density than in either of its constituent factors. However, it is by no means universally true that the conduction current-density remains even approximately constant; it is thus of interest to trace out in somewhat greater detail the consequences of the view outlined above, by working out an example where the change of conductivity with altitude is not absolutely sudden, as in the crude illustration cited, but takes place in a continuous manner.

In the problem considered, a distribution of potential-gradient over the surface of the earth is expressed in a series of Legendre coefficients in the form:

$$\left(\frac{\partial V}{\partial r}\right)_{r=a} = A_0 + A_1 P_1(\mu) + \cdots + A_n P_n(\mu) + \cdots \qquad (1)$$

r, θ, being the polar coördinates of a point, and θ being measured from the line joining the center of the earth to that of the sun.

It is shown that a solution of the equation of continuity, subject to $V = 0$ at the sphere of radius a (the radius of the earth), and giving the distribution (1) over this sphere is given by:

$V =$ any function of ψ,

where,

$$\psi = -\frac{\sigma_0 A_0 a^2}{r} + \frac{\sigma_0 A_1}{3}\left(r - \frac{a^3}{r^2}\right) P_1(\mu) + \cdots + \frac{\sigma_0 A_n}{2n+1}\left[\frac{r^n}{a^{n-1}} - \frac{a^{n+2}}{r^{n+1}}\right] P_n(\mu) + \cdots,$$

provided that σ the conductivity is given by:

$$\sigma = \left(\frac{dV}{d\psi}\right)^{-1}, \text{ while}$$

σ_0 is the conductivity at the surface of the earth. As a particular example, the case

$$\left(\frac{\partial V}{\partial r}\right)_{r=a} = X_0 + X_0 \alpha (1 + \cos\theta) + \cdots \qquad (2)$$

is considered, α being a constant which, for illustration, is taken as 0.25. This expression corresponds to a decrease in $(\partial V/\partial r)_{r=a}$ from $\theta = 0$ to $\theta = \pi$. Values of σ and $\partial V/\partial r$ given respectively by:

$$\sigma = \sigma_0 e^{\beta\left[1 - \left\{\frac{a}{r} - \frac{\alpha}{3(1+\alpha)}\left(\frac{r}{a} - \frac{a^2}{r^2}\right)\cos\theta\right\}\right]} \qquad (3)$$

and

$$\frac{\partial V}{\partial r} = X_0\left[(1+\alpha)\frac{a^2}{r^2} + \frac{\alpha}{3}\left(1 + \frac{2a^3}{r^3}\right)\cos\theta\right] e^{-\beta\left[1 - \left\{\frac{a}{r} - \frac{\alpha}{3(1+\alpha)}\left(\frac{r}{a} - \frac{a^2}{r^2}\right)\cos\theta\right\}\right]} \qquad (4)$$

satisfy the necessary conditions for this case, β being constant. It is shown that if β is chosen so as to make σ increase by a factor of 30 at an altitude of 10 kilometers, the conductivity at such an altitude as is given by $r = 2a$, for example, is practically infinite for all values of θ; and, the distribution of $\partial V/\partial r$ in the region $r < 2a$ is sensibly the same as would have resulted from any other expression for σ which was sensibly the same as that chosen in the region $r < 2a$, even though, as r increased in value, this other value of σ gradually departed from the form (3) in such a way as to avoid the peculiarities associated with that form for large values of r.

It will be observed that the form (3) is such that σ increases more rapidly with r when $\theta = 0$ than for any other value of θ, and it is thus of the type to represent qualitatively the influence of the sun's ultra-violet light. The constant α may be taken as representing the intensity of the sun's effect. If there were no ultra-violet light, α would be zero, and (3) and (4) would indicate equal rates of increase of σ and of decrease of $\partial V/\partial r$ with altitude for all values of θ. Moreover, the potential-gradient would be constant over the earth's surface, as shown by (2).

The views outlined in the paper are applicable to the case of the solar eclipse, and are in line with experiment in predicting a diminution of the potential-gradient as a result of removal of the sun's rays.

Mr. Bussey's paper on "Methods of finding the square root of a number by means of a computing machine" gave two methods. (1) If a is an approximate value of the square root of a number $N = (a + b)^2$, the number $\frac{1}{2}[(N/a) + a]$ is a better approximation. This follows from the fact that

$$\tfrac{1}{2}[(N/a) + a] = (a + b) + b^2/2a$$

which is nearly equal to $(a + b)$ if b is small in comparison with a. The divisions involved in this method can be performed with ease on a computing machine. The method is equivalent to Newton's method of approximation. (2) The square root of a number less than 10,000 can be found by repeated subtraction by means of the following theorems:

(i) a^2 = the sum of a consecutive odd numbers beginning with unity.

(ii) $2abt + b^2$ = the sum of b consecutive odd numbers beginning with $2\,at + 1$.

Example:

$$\left.\begin{array}{r} 529-100 = 429 \\ 429-300 = 129 \end{array}\right\}\ \text{Two subtractions; therefore the first figure of the square root is 2.}$$

$$\left.\begin{array}{r} 129-\ 41 = 88 \\ 88-\ 43 = 45 \\ 45-\ 45 = 0 \end{array}\right\}\ \text{Three subtractions; therefore the second figure of the square root is 3. Answer, 23.}$$

The method can be extended so as to apply to numbers greater than 10,000 and to numbers which are decimal fractions.

Mr. Beal's paper reviewed *An introductory treatise on dynamical astronomy*, by H. C. Plummer (see this MONTHLY, June, 1919, pages 253–254).

Miss Quigley gave numerous illustrations of success she has attained by popular lectures in interesting students in mathematics and its study.

The program was followed by an excellent dinner at which the Section was guest to Hamline University. For the ensuing year R. A. JOHNSON was elected chairman, R. M. BARTON, secretary-treasurer, C. H. GINGRICH, C. H. YEATON, R. M. MATHEWS, executive committee.

It was voted that resolutions upon the death of Father Etzel be drawn up and placed in the minutes of the Section and that a copy of the same be sent to St. Thomas College.

R. M. BARTON, *Secretary-Treasurer.*